

Testing the Reliability of Mathematical Models

By

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Dissertation submitted in partial fulfillment of
the requirement for the
Bachelor of Engineering (Hons)
(Electrical & Electronics Engineering)

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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
Electrical & Electronics Engineering Programme
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In partial fulfillment of the requirement for the
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(ELECTRICAL & ELECTRONICS ENGINEERING)

Approved by,



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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except s specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.


Balkan Arazbayev

ABSTRACT

The mathematical models are extensively used in the engineering fields. It is a very powerful tool to design, analyze and test the systems without actually building them, which of course reduces time spent on the designing. The question was raised, is it actually appropriate to use the mathematical models for testing purposes, how precise is the outcome and what kind of information about the system the mathematical model gives. In order to investigate this situation, testing the reliability of the mathematical models was proposed. The purpose of this project is to scrutinize the mathematical models, investigate the pros and cons, the criteria and limitations that a mathematical model have and of course the errors introduces if those criteria and limitations are violated. The scope of the project covers general mathematical models for sake of investigating how they are used and their limitations. Also, models related to electrical and electronics field are studied to quantify the errors. As this project is mostly research based, the theories behind mathematical models are investigated and considered and then MATLAB programming is used to illustrate how models are used and their outcomes for given inputs for different situations. The results of the MATLAB simulations show the quantity and percentage of the errors and are given in form of table later in the chapters. Considering what has been studied and the results of the simulations, mathematical models do not ideally represent real life models, although if carefully designed the errors could be reduced to acceptable limits and a close approximation of the real life model could be obtained. As for educational purposes the little percentage of the error does not carry any significance mathematical models can be used to enhance understanding of the student. If nature of the application requires high precision, extra care must be taken while modelling and designing.

ACKNOWLEDGEMENT

As this project requires deep knowledge of the systems, mathematical modeling and MATLAB programming the success of the project heavily relies on the guidance by someone with enormous experience. This project would not be complete without the kind assistance and guidance from Azizan Zainal Abidin, who was in fact interested in the outcome of the project as much as the author. The references and suggestion provided during literature review, not mentioning MATLAB programming, were considerably significant to the author's perspective and point of view. I would like to thanks from the bottom of my heart Azizan Zainal Abidin, whose assistance is highly appreciated and without which it would take the author much longer time and much more effort to complete the project.

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1. INTRODUCTION

1.1 BACKGROUND STUDY

Study of control systems is essential for students aiming for degrees in electrical, mechanical, aerospace, or chemical engineering. Control systems are found in a wide range of variety of applications within these disciplines, from aircraft and spacecraft to robots and process control systems.

Control systems are an essential part of modern society. Numerous applications are all around us: The rockets fire and the space shuttle lifts off to earth orbit; in splashing cooling water, a metallic part is automatically machined; a self-guided vehicle delivering material to workstations in an aerospace assembly plant glides along the floor seeking its destination. There are just a few examples of the automatically controlled systems that can be created.

We are not the only creators of automatically controlled systems; these systems also exist in nature. Within our own bodies are numerous control systems such as the pancreas, which regulates our blood sugar. In time of “fight or flight”, our adrenaline increases along with our heart rate, causing more oxygen to be delivered to our cells. Our eyes follow a moving object to keep it in view; our hands grasp the object and place it precisely at a predetermined location.

In designing a particular system, the mathematical modeling is used in order to analyze and test the model to have some clues of how the systems will behave in the real world. Since mathematical modeling is inevitable part of the design process, we are concerned about the precision of the results that are obtained using mathematical modeling.

1.2 PROBLEM STATEMENT

As we are concerned about the relevance of the mathematical models to the actual behavior of the systems in the real world we would like to investigate the mathematical models, their errors and if the errors could be eliminated, if errors could not be eliminated then how significant they are and how do they affect the outcome of the design. We would also like to know if the errors would make systems unstable, which of course is highly undesirable during designing process. Keeping in mind this issue, a study of mathematical models and errors was found necessary to justify the usage of mathematical models as reliable representation of the systems used.

1.3 OBJECTIVES

The objectives of this project are stated below:

- Investigate the outcome of the ideal model
- Compare the results from ideal model to theoretical values
- Find the criteria and limitations that mathematical model has
- Determine the error introduces if limitations are violated
- Use MATLAB in order to simulate and test a model
- Define pros and cons of using mathematical model
- Make conclusion about how appropriate is using the mathematical models

Besides general models of known laws and theories, electrical models will be considered in order to gain knowledge on how mathematical models are used in electrical field.

If the above objectives are satisfied, a whole new perspective on the usage of mathematical models will be gained.

As this project involves a lot of literature review and then some simulations of the models, the time given for completion can be considered enough.

1.4 SCOPE OF THE PROJECT

The scope of the project is to investigate general mathematical models used in engineering fields. Different applications of mathematical models in various fields are considered and the significance of the error that mathematical models introduce due to assumptions, simplification of complex models for ease of modeling, linearization and truncation errors are discussed with relevance to the cases mentioned. A more detailed study about errors and their quantity is conducted in order to visualize the effect and reliability of using a mathematical model. Several MATLAB simulations will be conducted to find out about weakness of the mathematical models and the results will be considered in concluding whether usages of models are appropriate in the engineering fields.

2. THEORY

Knowing the importance of mathematical modeling in the modern engineering fields we would also like to know the reliability of the mathematical models. In order to investigate this topic, models with different level of complexity, using MATLAB programming, will be tested for the results they give and then compared to the results that theoretically should have been obtained. As every system has its own limitations, the criteria for stable output will be identified, and then by violating those criteria the amount of the error introduced will be scrutinized. Also the significance of the deviation from the actually results is of concern, and graphs and plots will be given to provide assistance to visualize the amount of deviation and the effects.

To start a study on the errors of the mathematical models and their significance a deep knowledge of mathematical models is required. To see how mathematical models are used in various engineering fields a review of previously studied concepts where mathematical models are utilized was found to be necessary. A number of different cases where modeling is a part of the study were considered, and depending on the role that a model played in the study the reliability of the model and the significance of the errors were studied.

3. LITERATURE REVIEW

3.1 Computation of Model Parameters

It happens so that sometimes some parameters of the system that is being modeled are not available and engineers have to find some way of determining them. The parameter estimation refers to the process of comparing the input-output relationship of actual system, and then estimate the unknown parameters mathematically depending on the results obtained. As we have various systems and situations, different approaches have been developed in order to optimize the parameter search for engineers. These approaches are Black-box, White-box and Gray-box which are discussed below:

1) Black-box

In Black-box approach, the system is described mathematically depending on the parameter estimation derived from test results. In this case, the actual value of the parameters being estimated are not very important and the match that was derived from test results are considered satisfactory. However, there are some drawbacks of this approach. It may be difficult to derive parameter for similar models using results from different tests, while the difference of the test results could be just experimental errors or equipment fault.

2) White-box

In White-box approach, the model cannot be just estimated using test results. In this case, best fit of the model is not satisfactory so in order to determine a model with maximum possible accuracy the knowledge on internal structure of the system is also required. All the criteria, such as initial conditions, or transient response of the system has to be taken into consideration and added as parameters to the model. This would usually require someone with deep knowledge on the subject and large experience, because trial-and-error method of estimation will take considerable longer time. By knowing the topic in-side-out

the model can be quickly constructed and tested using appropriate test vectors. Including all the significant parameters that bring changes to the outcome will help to estimate them by having necessary special-case tests for every parameter holding others constant.

3) Gray-box

Gray-box approach is something in the middle of black-box and white-box approaches. Knowledge on the matter is still required, while some flexibility allow around parameters. There is no definite cut between black and white box approaches. So in gray-box approach the relevance of some parameters is arguable. But still, tests have to be done in order to compare input and output in order to determine the unknown parameters of the mathematical model. The major drawback of this approach is to decide which parameters should be included and which ones could be omitted.

3.2 An infectious disease model

The following model shows the rate at which infection spreads among the population, certain assumptions were made, but not discussed here.

$$\frac{ds}{dt} = -asi$$

$$\frac{dr}{dt} = bi$$

$$\frac{di}{dt} = asi - bi$$

Here, s , represents susceptible population, r refers to removed population, and the difference of susceptible and removed equals to infected population which is denoted as i . Since all of these show the rate of change, they are variable with time t .

The constants can be taken as

$$a = \frac{6.8}{11}$$

$$b = \frac{1}{11}$$

Same model could be used to determine spread of other infectious diseases with proper constants, but not all estimation will give correct results. Moreover, using this model accurately for a disease that follows some other model, will give results which are not relevant to the case. Without knowing the behavior of the disease it is difficult to tell whether above model should be used to determine its rate of spread, and a person without proper knowledge will not be able to arrive at correct conclusions.

3.3 The predator-prey model

In the following equations, $F(t)$ denotes number of Foxes and $R(t)$ denotes number of rabbits at any given time t . It is a simple formulation of predator-prey model.

$$\begin{aligned}\frac{dF}{dt} &= c[-1 + dR(t)]F(t) \\ \frac{dR}{dt} &= a[1 - bF(t)]R(t)\end{aligned}$$

with a, b, c, d positive constants. By looking at the model it could be derived that rate of change of foxes increase when there are more rabbits, and the rate of change of rabbits decrease when there are more foxes. This model could be used to get correct approximation of the existing animals with properly estimated constants. However, using same model for other couple of animals will be a failure if there are significant parameters that have been omitted.

3.4 Newton's Second Law

Newton's second law states that the force acting on an object is directly proportional to the product of its mass and acceleration: $F \propto ma$. Usually, it is written as $F = ma$. Newton's Universal Gravitational law states that two bodies, say the earth and an object moving about the earth follow more advanced model:

$$m \frac{d^2 r(t)}{dt^2} = - \frac{Gmm_e}{|r(t)|^2} \frac{r(t)}{|r(t)|}$$

Here, G is gravitational constant independent of other parameters, $r(t)$ is the distance from center of Earth to the object moving around it, while m and m_e are the masses of the object and earth, respectively. This is an accurate model for most situations, but there are exceptions when this model fails.

If an object is moving around Earth close to the surface, the distance is assumed to be equal to the radius of Earth. Then, the equation simplifies as follows.

$$m \frac{d^2 r(t)}{dt^2} = -mgk$$

with k the unit vector directly upward from the earth's surface at the location of the object. The gravitational constant $g = 9.8 \text{meters/second}^2$

However, it is a known fact that the Earth does not have a perfect spherical shape and the radius varies from equator to the poles. So having calculated gravitational constant using this equation will give a one value which should not be equal for equator and poles of the Earth. A simple, general assumption introduces a defect and give incorrect estimation.

3.5 Modeling with Difference Equation

In some cases, the rate of change of the variable is discrete so if the current value of the variable is a_n then the next estimation would be a_{n+1} . The relationship between current and next value could be in a form of

$$a_{n+1} = \alpha a_n + \beta$$

In other words, the next estimation of the current value is some constant times current value and some offset β . This model is common, e.g., it is used for modeling bank loans. However,

the model could have more parameters and offset β could change with every iteration. So we write the parameter β which is dependent on the iteration number:

$$a_{n+1} = \alpha_1 a_n + \alpha_2 a_n^2 + \beta_n$$

Here, β will have different values for every iteration and therefore offset will change with every step. This could correspond to, for example, a population model where the migration levels change every time step. In some cases, the next estimation depends on the previous two values and the model becomes:

$$a_{n+1} = a_n + a_{n-1}$$

Here we can see that in order to determine next value, we need to know the current value and previous value, only then next value can be calculated.

Finally, the model could have some unknown parameter and be dependent on the previous value, so the model becomes:

$$a_{n+1} = f(a_n; a_{n-1}; a_{n-M-1})$$

the nature of the model and the unknown parameters could be determined using any of the three approaches mentioned earlier because without knowing the next estimation can be far from the real value.

3.6 Modeling with Ordinary Differential Equations

Difference equation and ordinary differential equations may look similar but the main difference is that in ordinary differential equations the time is assumed to be continuous. One of the simplest differential equations which denotes the rate of change with time variable:

$$\frac{dx}{dt} = \alpha x$$

In other words words, the rate of change of the quantity x is small if the quantity of x is small and the rate of change is large if the quantity of x is large. If $\alpha > 0$ then we have exponential

growth and if $\alpha < 0$ then we have exponential decay. However, additional terms can be added that completely change the nature of $x(t)$. For example

$$\frac{dx}{dt} = \alpha_1 x + \alpha_2 x^2$$

Here, the rate of change is parabolic because apart from the fact that it depends on quantity of x it also depends on the square of the quantity of x and some constant values of α_1 and α_2 . The estimation of the rate will be much different for larger values of x while small values of x may give acceptable approximations.

Equation could also be modeling for cases where the rate of change is depended on the previous values. Then the equation will change be similar to the following:

$$\frac{dx}{dt} = f(x(t)) + g(x(t - \tau))$$

This is referred to as a delay differential equation.

3.7 Modeling with Partial Differential Equations

In the previous examples the behavior of the models was affected by single independent variable, whereas in reality sometimes the models could involve several independent variables. To estimate rate of change, we have to hold other variable constant and differential for one independent variable at a time.

$$\frac{\partial f}{\partial t} = \alpha \frac{\partial^2 f}{\partial t^2}$$

or

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

3.8 Richardson's Extrapolation

Sometimes, when discrete limited data is available about certain phenomenon, extrapolation can be used to determine values beyond the limits of the data. Of course, different approaches are used, and Richardson Extrapolation is one way of improving the accuracy of the model. The basic idea of extrapolating results with step sizes h and $2h$ to $h = 0$ can be summarized as follows

- Model becomes more precise as $h \rightarrow 0$, usually using $h = 0$ is not feasible
- Compute a solution in using step size h : $y(x_k; h)$
- Compute a solution in using step size $2h$: $y(x_k; 2h)$
- Relative error can be calculated by subtracting acquired values from each other and the behavior of the truncation error identified.

For a method of order p , the exact solution can be expressed as:

$$y(x_k) = y(x_k, h) + ch^p + O(h^q)$$

Where c is a constant and p and q are integers with $p < q$. Solving the problem with step sizes h and $2h$ yields

$$\begin{aligned} y(x_k) &= y(x_k, h) + ch^p + O(h^q) \\ y(x_k) &= y(x_k, 2h) + c(2h)^p + O(h^q) \end{aligned}$$

Subtracting these equations, we obtain the truncation error as:

$$ch^p = \frac{y(x_k, h) - y(x_k, 2h)}{2^{p-1}} + O(h^q)$$

Thus, adding the error term to $y(x_k; h)$, we obtain a better approximation of the exact solution $y(x_k)$, because

$$y(x_k) = y(x_k, h) + \frac{y(x_k, h) - y(x_k, 2h)}{2^{(p-1)}} + O(h^p)$$

The algorithm for improving accuracy of the model is listed below:

1. Compute y_k using a method of order p with step sizes h and $2h$.
2. Estimate the truncation error ch^p with Richardson extrapolation
3. Include error coefficient in the model previously identified

3.9 The Delicate Science of Error

100% precise results could be achieved theoretically, but however, in reality it is not feasible. The difficulty with having infinitely precise models is that they are hard to analyze, they are too complex to be solved, and the behavior cannot be understood my just looking at it, or with simple analysis algorithms. Therefore, it is a common practice when the models are being reduced in order, or only first few coefficients are considered and the rest are ignored, or best approximation is taken if the original model introduces too many parameters that have to be taken into account. Usually simplification takes place for application purposes while most terms are kept during modeling. The important question to ask is would our results derived using mathematical modeling meet the requirements of the original equation. The dilemma is to decide how to simplify a model while keeping the accuracy as high as possible. An error analysis for a model was conducted to see what could be the deviations of the simplified model compared to original equation.

Following illustrates this concept by examining junior high school problem

$$\epsilon x^2 + x + 1 = 0$$

This equation can be solved using quadratic formula:

$$x_{1,2} = -\frac{1}{2\epsilon} \pm \frac{\sqrt{1-4\epsilon}}{2\epsilon}$$

For analysis sake, let us assume that the quadratic term was an unknown parameter e.g.,

$$\mathcal{F}(x) + x + 1 = 0$$

and that actually computing $f(x)$ might be rather expensive. We might take into consideration the fact that if the value of ϵ is very small, or very close to 0 then this term could safely be ignored. Now let us assume that $f(x)$ has been estimated to be $f(x) = x^2$. If ϵ is taken as zero

then clearly it follows that $x = -1$ is the unique solution. However, we know from our quadratic equation however that if $\varepsilon = 0.0000001$ (any non-zero number would do), then there are two solutions rather than one. So ignoring ε because the value is small, we will actually lose one solution for equation and get only one value of the root whereas it is supposed to be two. In addition, due to our ignorance and desire to avoid determining the value of $ef(x)$ for simplification sakes, not only we got one solution instead of two, but we got deviated answer because existence of second root changes the value of the first root as well. The inaccuracies introduced have to be explored.

Our main concern is how solutions change when ε differs from 0-value. In order to have more precise estimation of ε we will expand it as series:

$$x = a_0 + a_1\varepsilon + a_2\varepsilon^2 + a_3\varepsilon^3 \dots$$

Substituting this expansion into the original quadratic results in the new equation

$$\alpha_0 + 1 + (\alpha_0^2 + \alpha_1)\varepsilon + (2\alpha_0\alpha_1 + \alpha_2)\varepsilon^2 + \dots = 0$$

Setting the coefficients of the ε to zero gives the series solution for x as

$$x = -1 - \varepsilon - 2\varepsilon^2 + \dots$$

From the above equation we can see that as the value of ε changes, regardless toward positive or towards negative value, the value of x will change too. From this we can quantify the error introduce for different values of ε and calculate relative error in terms of percentage.

3.10 DC Motor Theory

Current flowing in a conductor creates a magnetic field around it. If we have a wire looped around cylinder, the magnetic field created would be in direction parallel to the axis of the cylinder. Using the principle of magnetic fields of a current-carrying conductors and the attraction/repulsion of magnets, a basic motor can be created. Magnets and loops of wire are configured in such a way that they magnetic activity among them is always repulsive (or attractive) which creates torque to rotate the stator of the motor.

The basic components of a DC electric motor include:

- **Frame** – Makes up the outer structure of the machine. It is the “skeletal system” of the motor and every other components are attached to it to hold it as a whole
- **Fields** – Are coils wire that generate a stationary magnetic field.
- **End shields** – Also called bearing housings, they serve the purpose of holding the armature centered in the frame.
- **Brushes** – Are used to provide DC to the armature. The brushes ride on the commutator.
- **Commutator** – Consists of many copper bars that are separated by mica. Each bar is connected to coils in the armature.
- **Armature** – Is the rotating portion of the motor that contains coils.

In a DC motor, two separate power supplies are need for armature and the stator, respectively. The power supplied to stator, will create a constant magnetic field. The power supplied to armature will create another magnetic fields which opposes to the magnetic field created by stator and thus create a torque which will turn the armature. Once the armature turns, the magnetic fields will not be opposing any more, but in fact, the polarity of the armature power supply will change, therefore the direction of the magnetic field created by armature will change and still be opposite to the field created by stator. Thus, rotation will continue with polarity of armature changing when the fields do not oppose each other. There are different types of windings that can the motor can be configured by.

Winding Types:

The three basic winding types that can be used to identify the type of DC motor include:

1. **Series:** Series configuration of armature and stator windings gives high initial torque, so this configuration essentially used for application where strong starting torque is required. As shown in the Figure 1. the armature winding and field windings are labeled A1, A2, and S1, S2 respectively. Armature and field windings are in series which also gives very low basic resistance because field windings consist of relatively few wire turns.

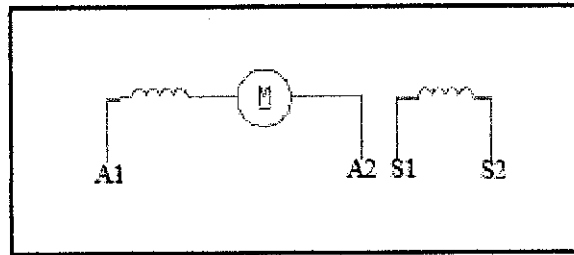


Figure 1: Series Motor

2. **Shunt:** Also known as parallel configuration, which gives a motor the ability to operate at constant speed. As can be seen in the Figure2, armature windings are labeled A1 and A2, while parallel field windings are labeled F1 and F2. Shunt connected motors are normally used as crane and machine tool motors and have a relatively high basic resistance.

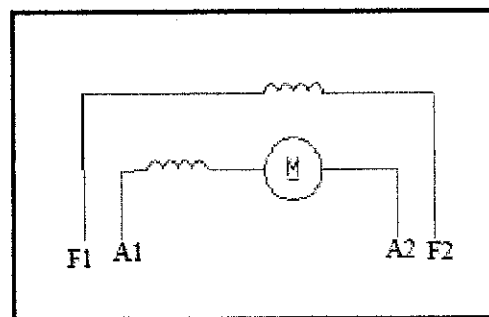


Figure 2: Shunt Wound Motor

3. **Compound:** As the name suggests, in this configuration, both series and shunt windings are utilized together in order to combine the benefits of both configurations. This type of configuration give high initial torque and a constant operation speed. The basic resistance changes with the speed of operation. The configuration can be seen in the Figure3, where armature windings are labeled A1 and A2, series field windings are labeled S1 and S2, and parallel windings are labeled F1 and F2. Compound motors are the most common and are commonly found in industrial manufacturing.

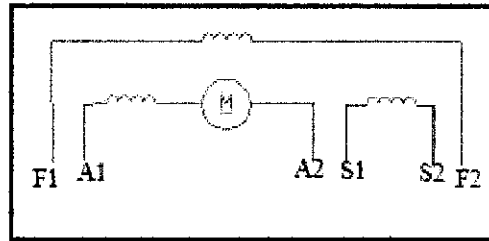


Figure 3: Compound Wound Motor

As it can be seen, there are few coils to compare to each other in an assembled DC machine. However, procedures can be developed for winding testing that provide a high level of test result confidence.

3.11 Common DC Motor Electrical Faults

There are various cases which lead to the failure of the motor, usually they arise due to temperature, friction and internal contamination and most common faults that occur will be described below:

1. **Winding Contamination:** winding contamination occurs due to carbon dust that is trapped in stationary and rotation windings of the motor. The problem is further worsened by the maintenance, as they use compressed air to clean the motor, which tightly packs the carbon dust in the corners and let them accumulate. These carbon packs may create path for electricity between windings and result in poor operation, or if the path is created between winding and ground, a short circuit will be created which will increase the current infinitely and essentially burn the windings.
2. **Temperature related faults:** Temperature is the greatest enemy of the electrical equipment and should be given a deep consideration since it is one of the most common faults of the DC motors. A poor ventilation of the DC motors can lead to increase of temperature, which reduces the reliability of the motor. Another problem with temperature is that it destroys the insulation, the life span of the insulations are halved for every 10 °C. As insulation is destroyed, short circuit may occur in the windings which, as was mentioned before may lead to high current that burns the windings and makes a motor

useless. In addition to the insulation system degrading, brushes also degrade faster, causing increased wear on the commutator and additional carbon contamination of the windings. Another fault that is related to heat is generated from applications that require the fields to be energized while armature is at rest. For this common mode of operation, motor needs separate blower to provide cooling to the motor that normally has filters that must be kept clean. This type of fault normally results in shorted shunt coils, reducing the motor's ability to produce torque and may end with the dangerous condition of armature over speeding if not maintained properly.

3.12 Linearization

Sometimes, if the given equations contain non-linear components, the equation should be linearized first, and then considered for further computations. Linear approximation of the function is determined in order to obtain transfer function for systems containing non-linear components. First step of linearization is to recognize the non-linear component and write the respective non-linear differential equation. Usually, linearization is done for small inputs which are not much far from equilibrium point, or steady-state point. For example, when pendulum is at rest, it is in equilibrium. The angular displacement is described by a non-linear differential equation, but it can be expressed with a linear differential equation for small deviations about the equilibrium point. In mathematical terms it looks like as follows:

$$[f(x) - f(x_0)] \approx m_a(x - x_0)$$

$$\delta f(x) \approx m_a \delta x$$

$$f(x) \approx f(x_0) + m_a \delta x$$

Lets linearize $f(x) = 5 \cos(x)$ about $x = \pi/2$

First we take the derivative of $f(x)$

$$f'(x) = (5 \cos(x))' = (-5 \sin(x))$$

At point $\sin(x) = 1$, $m_a = -5$

$$f(x_0) = f(\pi/2) = 5 \cos(\pi/2) = 0$$

so $f(x) = 0 + (-5)\delta x$ at point π because that where $\sin(x) = 0$

4 METHODOLOGY

As this is a research based project, most of the analysis is done by reading other related topics in order to gather information on how to go about the project. Different topics were covered, namely, mathematical modeling in science, concept of errors in mathematical models, different sources of errors and how to quantify the errors. Literature review was quite fruitful because the ideas obtained directed the author during the project. Making use of available concepts on how to improve mathematical models and how to reduce errors helped author to get a concise perception of the topic. Gathering all the information and finding common ground in them, and then manipulating the data by analysis gave expected results.

Once the general knowledge of mathematical models have been obtained, further analysis was conducted by using MATLAB programming to simulate some models to get the results for given inputs. Comparing results obtained by giving different inputs gave a chance of quantifying the error and to see how significant is the deviation from the original results. Graphs, plots and tables are used to help to visualize the differences. Electrical motor models are considered for their performance under different circumstances and cases where input parameter change or have no effect on the outcome of the model are given. Using two methods mentioned above, information gathering and simulation, the project can be finished successfully by indicating the deviations of the results and deciding whether they are significant or not.

5 RESULTS AND DISCUSSION

5.1 Ideal Spring

For Ideal Spring it is assumed that there are no losses, that the spring could be stretched infinitely. Using Hook's Law $F = -k \times x$ we can calculate the values of F for different values of stretching distance x

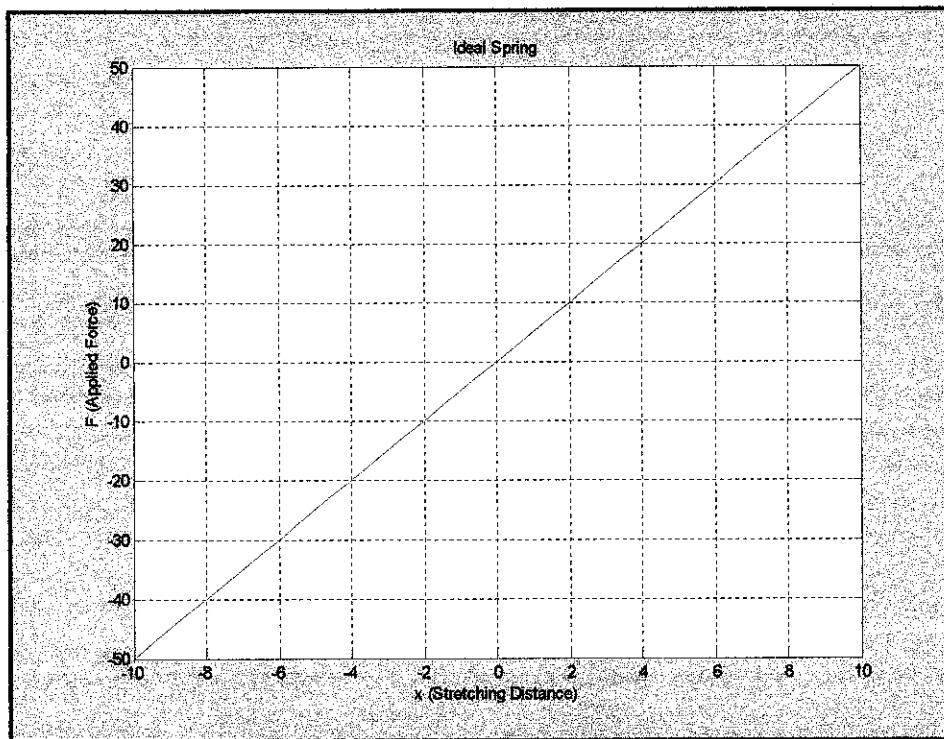


Figure 4: Plot for Ideal Spring

This plot shows the relevance between the stretching distance x and the applied force F respectively. This is ideal case, where spring is assumed to have infinite stretching distance.

5.2 Spring with limited stretching distance

As we all know the springs could not be stretched infinitely, and that they have a finite distance, after which the spring will undergo inelastic deformation and would be damaged permanently. If the spring undergoes inelastic deformation, it would no longer follow the

Hook's Law, and any calculations made without considering the limited stretching distance would lead to an error, if the stretching distance is significant in the calculations.

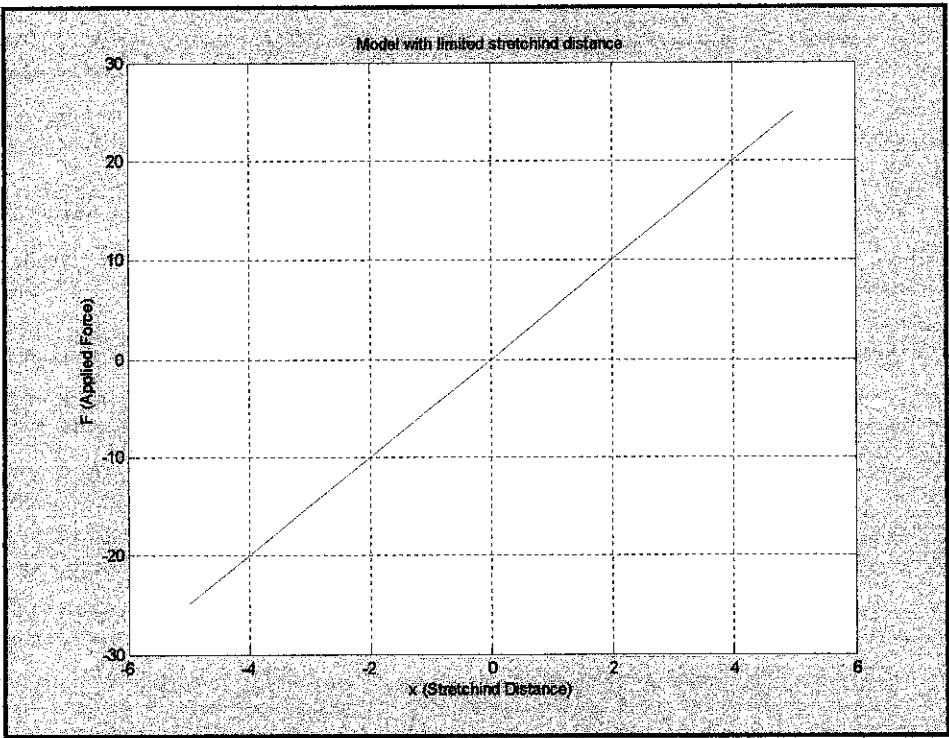


Figure 5: Model with limited stretching distance

As we can see on the plot, the F is defined only for the values of x between -5 and 5 , and 0 otherwise. If the applied force is calculated ignoring the stretching limit, this would lead to a significant error in the calculations since there will be no force applied by spring if it goes into inelastic deformation region and our expectations would not be met.

5.3 Spring with different stiffness constant k

In this case we are considering the fact that we can never measure the actual values of the components precisely, the general practice is to assume that the value is equal to the closest acceptable value. This leads to an insignificant error on small scale calculations, but on the large scale, error might go beyond acceptable level of accuracy.

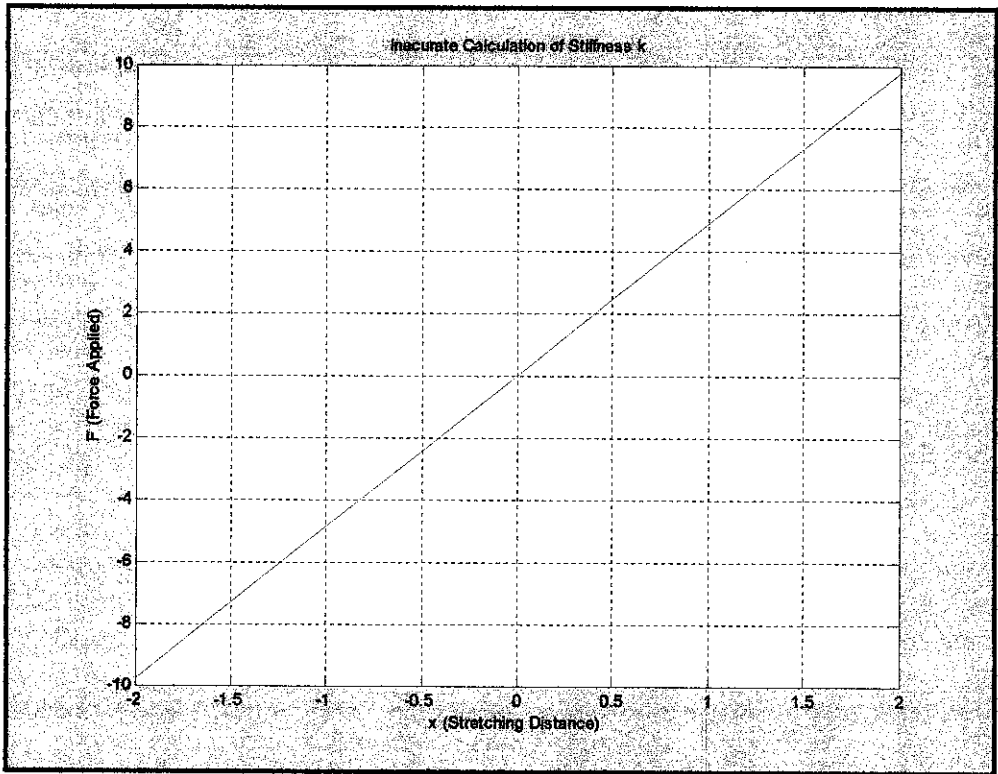


Figure 6: Inaccurate Calculation of Stiffness k

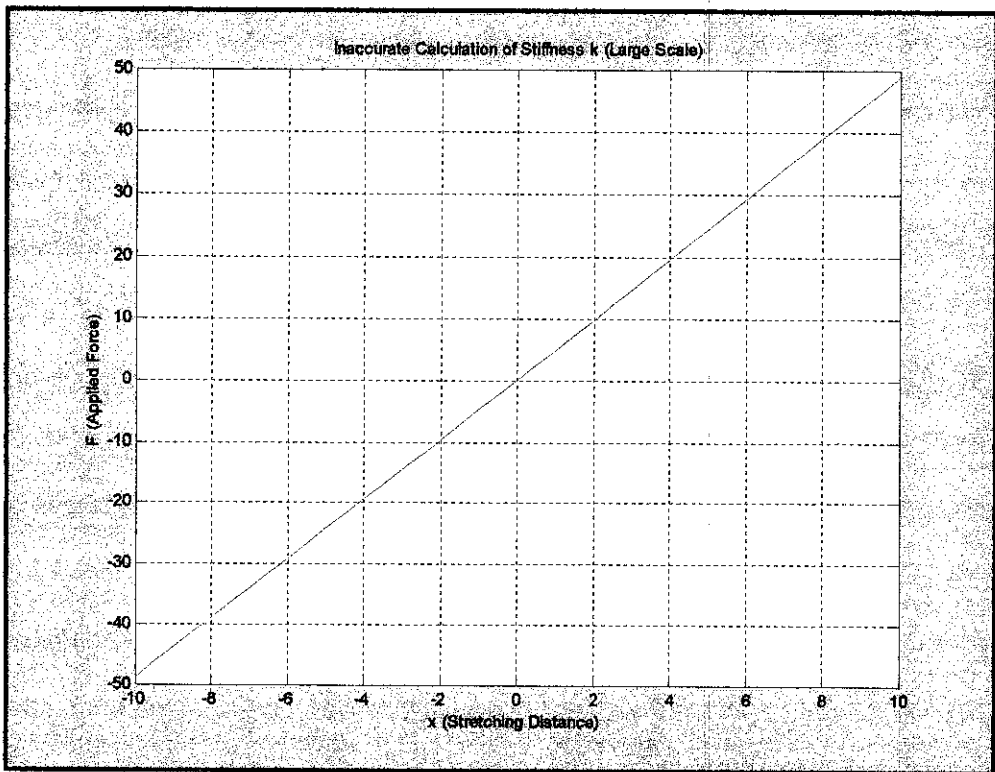


Figure 7: Inaccurate Calculation of Stiffness k (Large Scale)

If we compare the two plots above to the plot of ideal case, we will see that for small values of x the applied force would be more or less the same, but as values of x increase, the deviation from the ideal case also increases. Yet again, if the calculations are made based on approximate value of stiffness k , ignoring the fact that in the real life it could be different would lead us to the wrong conclusions, which may drive the system into unstable condition.

5.4 Truncation Error

Here is another simulation done using Microsoft Excel. A random equation was used, $2X^5+3X^4+0.1X^3+0.01X^2+0.001X$ in order to calculate results for different values of X , results for the equation omitting the last part, $2X^5+3X^4+0.1X^3+0.01X^2$ and then the relative error in percentage and another set of results for $2X^5+3X^4+0.1X^3$ with relative error given as shown in Table 1. It is interesting to notice that as value of X grows the differences of results become more significant, while the percentage of relative reduces. For this particular model, it is safe to say that for large values of X this model can be considered reliable, even with the truncated parts. As can be seen from the table, the difference of results is little, and the percentage of relative error is always less than 0 (for medical drug testing, percentage of safety is set to 99%, which is 1% of error). This example only illustrates the truncation error, another simulation and be run to display the case where model chosen does not represent the phenomenon under study completely. Let say that the model which was chosen to be $2X^5+3X^4+0.1X^3+0.01X^2+0.001X$ is in fact should be represented by equation $2.1X^5+3.02X^4+0.13X^3+0.012X^2+0.0095X$. In this case, it is interesting to notice how percentage of error first decreases and than increases again around point of $x=1.0$. Judging from the Table 2, it is obvious how the results differ, and it helps visually to decide whether to use the model or not, with percentage of relative error given, anybody can decide if it is appropriate for him or not.

Table 1: Truncation Error

X	$2X^5+3X^4+0.1X^3+0.01X^2+0.001X$	$2X^5+3X^4+0.1X^3+0.01X^2$	% Error	$2X^5+3X^4+0.1X^3$	%Error
0.100	0.0006200	0.0005200	16.12903226	0.0004200	19.23076923
0.200	0.0068400	0.0066400	2.923976608	0.0062400	6.024096386
0.300	0.0330600	0.0327600	0.907441016	0.0318600	2.747252747
0.400	0.1056800	0.1052800	0.378501136	0.1036800	1.519756839
0.500	0.2655000	0.2650000	0.188323917	0.2625000	0.943396226
0.600	0.5701200	0.5695200	0.105241002	0.5659200	0.632111252
0.700	1.0963400	1.0956400	0.063848806	1.0907400	0.447227191
0.800	1.9425600	1.9417600	0.041182769	1.9353600	0.329597891
0.900	3.2311800	3.2302800	0.027853601	3.2221800	0.250752257
1.000	5.1110000	5.1100000	0.019565643	5.1000000	0.195694716
1.100	7.7596200	7.7585200	0.014175952	7.7464200	0.15595758
1.200	11.3858400	11.3846400	0.010539407	11.3702400	0.126486213
1.300	16.2320600	16.2307600	0.008008842	16.2138600	0.104123282
1.400	22.5766800	22.5752800	0.006201089	22.5556800	0.086820629
1.500	30.7365000	30.7350000	0.004880191	30.7125000	0.073206442
1.600	41.0691200	41.0675200	0.003895871	41.0419200	0.062336367
1.700	53.9753400	53.9736400	0.003149586	53.9447400	0.053544856
1.800	69.9015600	69.8997600	0.00257505	69.8673600	0.04635209
1.900	89.3421800	89.3402800	0.002126655	89.3041800	0.040407306
2.000	112.8420000	112.8400000	0.00177239	112.8000000	0.035448423
2.100	140.9986200	140.9965200	0.001489376	140.9524200	0.031277368
2.200	174.4648400	174.4626400	0.001260999	174.4142400	0.027742329
2.300	213.9510600	213.9487600	0.001075012	213.8958600	0.024725546
2.400	260.2276800	260.2252800	0.000922269	260.1676800	0.022134667
2.500	314.1275000	314.1250000	0.000795855	314.0625000	0.019896538
2.600	376.5481200	376.5455200	0.000690483	376.4779200	0.017952677
2.700	448.4543400	448.4516400	0.000602068	448.3787400	0.016255933
2.800	530.8805600	530.8777600	0.000527426	530.7993600	0.014767995
2.900	624.9331800	624.9302800	0.00046405	624.8461800	0.013457501
3.000	731.7930000	731.7900000	0.000409952	731.7000000	0.01229861

Table 2: Miscalculation of coefficients

X	$2X^6+3X^4+0.1X^3+0.01X^2+0.001X$	$2.1X^6+3.02X^4+0.13X^3+0.012X^2+0.0095X$	Relative Error
0.100	0.0006200	0.0015230	59.29087328
0.200	0.0068400	0.0089240	23.35275661
0.300	0.0330600	0.0370050	10.66072152
0.400	0.1056800	0.1128560	6.358545403
0.500	0.2655000	0.2783750	4.625056129
0.600	0.5701200	0.5927880	3.823964048
0.700	1.0963400	1.1351690	3.420547954
0.800	1.9425600	2.0069600	3.20883326
0.900	3.2311800	3.3344910	3.098253976
1.000	5.1110000	5.2715000	3.044674191
1.100	7.7596200	8.0016530	3.024787503
1.200	11.3858400	11.7410640	3.025483891
1.300	16.2320600	16.7408150	3.03900975
1.400	22.5766800	23.2894760	3.06059269
1.500	30.7365000	31.7156250	3.087200709
1.600	41.0691200	42.3903680	3.116859
1.700	53.9753400	55.7298590	3.148256664
1.800	69.9015600	72.1978200	3.180511545
1.900	89.3421800	92.3080610	3.213024917
2.000	112.8420000	116.6270000	3.245389147
2.100	140.9986200	145.7761830	3.277327545
2.200	174.4648400	180.4348040	3.308654355
2.300	213.9510600	221.3422250	3.339247629
2.400	260.2276800	269.3004960	3.369030557
2.500	314.1275000	325.1768750	3.397958419
2.600	376.5481200	389.9063480	3.426009366
2.700	448.4543400	464.4941490	3.453177835
2.800	530.8805600	550.0182800	3.47946981
2.900	624.9331800	647.6320310	3.504899374
3.000	731.7930000	758.5665000	3.529486209

5.5 Linearization Error

In some cases, when modeling with non-linear models becomes unfeasible, linearization is applied to simplify the equation. But it should be applied with extra care, because linearization has limited approximation region. As in be seen in Figure 8, linearization was applied for cosine function at point 5.5. It might give close approximation for values close to 5.5 whereas moving far away from point 5.5 will give totally different results. Also, it should be noticed that to the right of the point 5.5, the sinusoid changes and starts to decrease while the line drawn for approximation keeps increasing. This suggests that the limits where linearization should be properly calculated in order to avoid significant errors of the approximation.

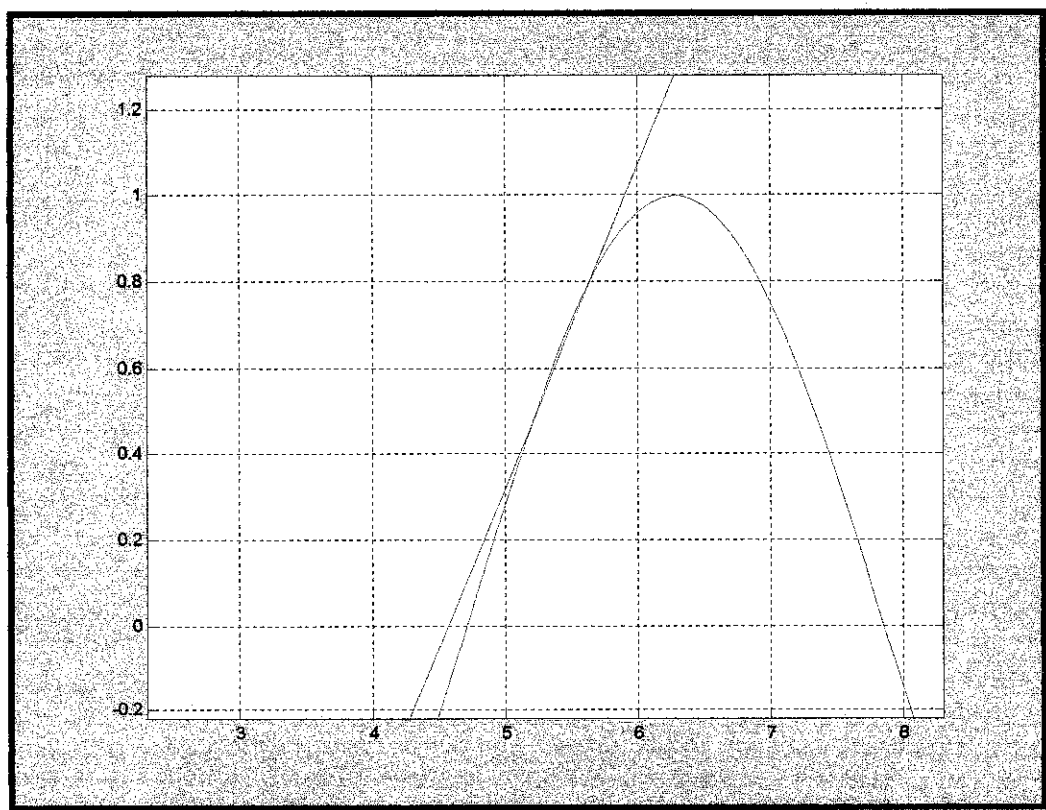


Figure 8: Linearization Error

As can be seen in Table 3, the values of x , $\cos(x)$, linearized values of $\cos(x)$ and the percentage errors are given. This table shows how error first reduced and then starts increasing again depending on the values of x . the table clearly shows that beyond values of $x < 5$ and for $x > 6$ the

error will be growing and introduce significant changes. Therefore, linearization, in this case, can only be used for values of $5 < x < 6$, assuming that 5% error is acceptable.

Table 3: Linearization Percentage Error

X	Cos(x)	Linearized Values	Percentage Error
5	0.283662185	0.27	4.816358
5.1	0.377977743	0.37	2.110638
5.2	0.468516671	0.46	1.817795
5.3	0.554374336	0.545	1.690976
5.4	0.634692876	0.62	2.314958
5.5	0.708669774	0.695	1.928934
5.6	0.775565879	0.77	0.717654
5.7	0.834712785	0.82	1.762616
5.8	0.885519517	0.86	2.88187
5.9	0.927478431	0.9	2.962703
6	0.960170287	0.92	4.183663

5.6 Testing a DC motor model

5.6.1 Calculation of Torque

Now, let us consider a simple model for torque in an electrical motor. Torque is given by the equation $T=2BilNr\sin\theta$ where:

T = torque

B = magnetic flux (taken as 50 for this experiment)

i = current in the conductors (taken as 0.03 for this experiment)

l = length of the loops of conductors (taken as 0.5 for this experiment)

N = number of loops (taken as 100 for this experiment)

r = radius of the loops (taken as 0.1 for this experiment)

At first glance, this model looks pretty simple, because everything is hold constant except for the values of $\sin\theta$. In the following table, we can see how torque changes with the values of $\sin\theta$ as it is the only variable parameter. But this of course is not the case, as we never have a negative torque since current reverses after every π rotation. So in our case, we have to take the variation from 0 to 1, as that will give the variation of the torque.

Table 4: Values of Torque (with negative values)

<i>B</i>	<i>l</i>	<i>r</i>	<i>N</i>	<i>l</i>	<i>sin(theta)</i>	<i>torque</i>
50	0.03	0.1	100	0.5	-1.000	-15.000
50	0.03	0.1	100	0.5	-0.950	-14.250
50	0.03	0.1	100	0.5	-0.900	-13.500
50	0.03	0.1	100	0.5	-0.850	-12.750
50	0.03	0.1	100	0.5	-0.800	-12.000
50	0.03	0.1	100	0.5	-0.750	-11.250
50	0.03	0.1	100	0.5	-0.700	-10.500
50	0.03	0.1	100	0.5	-0.650	-9.750
50	0.03	0.1	100	0.5	-0.600	-9.000
50	0.03	0.1	100	0.5	-0.550	-8.250
50	0.03	0.1	100	0.5	-0.500	-7.500
50	0.03	0.1	100	0.5	-0.450	-6.750
50	0.03	0.1	100	0.5	-0.400	-6.000
50	0.03	0.1	100	0.5	-0.350	-5.250
50	0.03	0.1	100	0.5	-0.300	-4.500
50	0.03	0.1	100	0.5	-0.250	-3.750
50	0.03	0.1	100	0.5	-0.200	-3.000
50	0.03	0.1	100	0.5	-0.150	-2.250
50	0.03	0.1	100	0.5	-0.100	-1.500
50	0.03	0.1	100	0.5	-0.050	-0.750
50	0.03	0.1	100	0.5	0.000	0.000
50	0.03	0.1	100	0.5	0.050	0.750
50	0.03	0.1	100	0.5	0.100	1.500
50	0.03	0.1	100	0.5	0.150	2.250
50	0.03	0.1	100	0.5	0.200	3.000
50	0.03	0.1	100	0.5	0.250	3.750
50	0.03	0.1	100	0.5	0.300	4.500
50	0.03	0.1	100	0.5	0.350	5.250
50	0.03	0.1	100	0.5	0.400	6.000
50	0.03	0.1	100	0.5	0.450	6.750
50	0.03	0.1	100	0.5	0.500	7.500
50	0.03	0.1	100	0.5	0.550	8.250
50	0.03	0.1	100	0.5	0.600	9.000
50	0.03	0.1	100	0.5	0.650	9.750
50	0.03	0.1	100	0.5	0.700	10.500
50	0.03	0.1	100	0.5	0.750	11.250
50	0.03	0.1	100	0.5	0.800	12.000
50	0.03	0.1	100	0.5	0.850	12.750
50	0.03	0.1	100	0.5	0.900	13.500
50	0.03	0.1	100	0.5	0.950	14.250
50	0.03	0.1	100	0.5	1.000	15.000

In the following table, we can see modified values where $\sin\theta$ is allowed to change only from 0 to 1.

Table 5: Values of torque (non-negative values)

B	i	r	N	l	$\sin(\theta)$	torque
50	0.03	0.1	100	0.5	0.000	0.000
50	0.03	0.1	100	0.5	0.025	0.375
50	0.03	0.1	100	0.5	0.050	0.750
50	0.03	0.1	100	0.5	0.075	1.125
50	0.03	0.1	100	0.5	0.100	1.500
50	0.03	0.1	100	0.5	0.125	1.875
50	0.03	0.1	100	0.5	0.150	2.250
50	0.03	0.1	100	0.5	0.175	2.625
50	0.03	0.1	100	0.5	0.200	3.000
50	0.03	0.1	100	0.5	0.225	3.375
50	0.03	0.1	100	0.5	0.250	3.750
50	0.03	0.1	100	0.5	0.275	4.125
50	0.03	0.1	100	0.5	0.300	4.500
50	0.03	0.1	100	0.5	0.325	4.875
50	0.03	0.1	100	0.5	0.350	5.250
50	0.03	0.1	100	0.5	0.375	5.625
50	0.03	0.1	100	0.5	0.400	6.000
50	0.03	0.1	100	0.5	0.425	6.375
50	0.03	0.1	100	0.5	0.450	6.750
50	0.03	0.1	100	0.5	0.475	7.125
50	0.03	0.1	100	0.5	0.500	7.500
50	0.03	0.1	100	0.5	0.525	7.875
50	0.03	0.1	100	0.5	0.550	8.250
50	0.03	0.1	100	0.5	0.575	8.625
50	0.03	0.1	100	0.5	0.600	9.000
50	0.03	0.1	100	0.5	0.625	9.375
50	0.03	0.1	100	0.5	0.650	9.750
50	0.03	0.1	100	0.5	0.675	10.125
50	0.03	0.1	100	0.5	0.700	10.500
50	0.03	0.1	100	0.5	0.725	10.875
50	0.03	0.1	100	0.5	0.750	11.250
50	0.03	0.1	100	0.5	0.775	11.625
50	0.03	0.1	100	0.5	0.800	12.000
50	0.03	0.1	100	0.5	0.825	12.375
50	0.03	0.1	100	0.5	0.850	12.750
50	0.03	0.1	100	0.5	0.875	13.125
50	0.03	0.1	100	0.5	0.900	13.500
50	0.03	0.1	100	0.5	0.925	13.875
50	0.03	0.1	100	0.5	0.950	14.250
50	0.03	0.1	100	0.5	0.975	14.625
50	0.03	0.1	100	0.5	1.000	15.000

The assumptions made in this experiment are as follows:

- B is taken constant as 50
- i is taken constant as 0.03

Other parameters such as l , N and r are indeed constant and not environment-dependent. But as we know, the value of resistance in the wire changes with temperature, which effectively will change the value of current in the conductors, and the magnetic flux B created by current will also change. Let us consider the situation where current i changes from 0.03 to 0.04 and B changes from 50 to 52.

Table 6: Variation of parameters B and i

B	i	R	N	l	$\sin(\theta)$	torque
52	0.04	0.1	100	0.5	0.000	0.000
52	0.04	0.1	100	0.5	0.025	0.520
52	0.04	0.1	100	0.5	0.050	1.040
52	0.04	0.1	100	0.5	0.075	1.560
52	0.04	0.1	100	0.5	0.100	2.080
52	0.04	0.1	100	0.5	0.125	2.600
52	0.04	0.1	100	0.5	0.150	3.120
52	0.04	0.1	100	0.5	0.175	3.640
52	0.04	0.1	100	0.5	0.200	4.160
52	0.04	0.1	100	0.5	0.225	4.680
52	0.04	0.1	100	0.5	0.250	5.200
52	0.04	0.1	100	0.5	0.275	5.720
52	0.04	0.1	100	0.5	0.300	6.240
52	0.04	0.1	100	0.5	0.325	6.760
52	0.04	0.1	100	0.5	0.350	7.280
52	0.04	0.1	100	0.5	0.375	7.800
52	0.04	0.1	100	0.5	0.400	8.320
52	0.04	0.1	100	0.5	0.425	8.840
52	0.04	0.1	100	0.5	0.450	9.360
52	0.04	0.1	100	0.5	0.475	9.880
52	0.04	0.1	100	0.5	0.500	10.400
52	0.04	0.1	100	0.5	0.525	10.920
52	0.04	0.1	100	0.5	0.550	11.440
52	0.04	0.1	100	0.5	0.575	11.960
52	0.04	0.1	100	0.5	0.600	12.480
52	0.04	0.1	100	0.5	0.625	13.000
52	0.04	0.1	100	0.5	0.650	13.520
52	0.04	0.1	100	0.5	0.675	14.040
52	0.04	0.1	100	0.5	0.700	14.560
52	0.04	0.1	100	0.5	0.725	15.080
52	0.04	0.1	100	0.5	0.750	15.600
52	0.04	0.1	100	0.5	0.775	16.120
52	0.04	0.1	100	0.5	0.800	16.640
52	0.04	0.1	100	0.5	0.825	17.160
52	0.04	0.1	100	0.5	0.850	17.680
52	0.04	0.1	100	0.5	0.875	18.200
52	0.04	0.1	100	0.5	0.900	18.720
52	0.04	0.1	100	0.5	0.925	19.240
52	0.04	0.1	100	0.5	0.950	19.760
52	0.04	0.1	100	0.5	0.975	20.280
52	0.04	0.1	100	0.5	1.000	20.800

If we compare tables 5 and 6, we can see how the maximum value of torque has changed from 15 to 20.8 that gives us $\frac{(15 - 20.8)}{15} \times 100 = -38\%$ error, which is quite big. And this is only considering the changes due to variation of temperature. If human error and equipment failure are taken into consideration too then the experimental results, which are expected as in table 5 will be much more different from it. After these tests, we can conclude that this model is not reliable for estimation of the torque, in order to have better estimations; parameters have to be calculated more precisely which will always include human error factor and equipment errors. This model can be used for simulation purposes; in order to use it for designing purposes a statistical data should be considered for the behavior of the model under different circumstances.

5.6.2 Calculation of voltage

Let us consider another mathematical model used in electrical motors which is as follows:

$$V_{applied} = EMF + I_a R_a$$

Where

$V_{applied}$ = voltage applied to the motor

EMF = Electromotive Force (voltage consumed by the motor)

I_a = Armature Current

R_a = Armature Resistance

EMF will be essentially constant, as voltage consumed by motor does not change with temperature or by any other ambient conditions. Another factor in this model is the product of the I_a and R_a , which may lead to confusion that it may alter the outcome of the estimation if temperature is varied. However, in this case, this does not have any effect as we know that I_a and R_a are inversely proportional and increase R_a will lead to decrease in I_a and the product will essentially be the same as the voltage consumed by armature is equal to

$$V_{armature} = I_{armature} \times R_{armature}$$

from Ohm's Law.

In the following table, the test data is given for armature resistance changing from 10Ω to 15Ω

Table 7: Testing for Applied Voltage

<i>EMF</i>	<i>R_a</i>	<i>I_a</i>	<i>V_{applied}</i>
50	10.00	2	70
50	10.15	1.970443	70
50	10.30	1.941748	70
50	10.45	1.913876	70
50	10.60	1.886792	70
50	10.75	1.860465	70
50	10.90	1.834862	70
50	11.05	1.809955	70
50	11.20	1.785714	70
50	11.35	1.762115	70
50	11.50	1.73913	70
50	11.65	1.716738	70
50	11.80	1.694915	70
50	11.95	1.67364	70
50	12.10	1.652893	70
50	12.25	1.632653	70
50	12.40	1.612903	70
50	12.55	1.593625	70
50	12.70	1.574803	70
50	12.85	1.55642	70
50	13.00	1.538462	70
50	13.15	1.520913	70
50	13.30	1.503759	70
50	13.45	1.486989	70
50	13.60	1.470588	70
50	13.75	1.454545	70
50	13.90	1.438849	70
50	14.05	1.423488	70
50	14.20	1.408451	70
50	14.35	1.393728	70
50	14.50	1.37931	70
50	14.65	1.365188	70
50	14.80	1.351351	70
50	14.95	1.337793	70
50	15.00	1.333333	70

The product of the I_a and R_a will always be equal to 20V in this case, no matter how much they change among themselves. The variation of temperature and therefore the variation of the armature resistance will have no effect on the outcome of the model. Therefore, this model is considered reliable, as changing input parameters have given same output results and did not introduce any errors.

As it can be seen from above two experiments, the model cannot be judged for reliability just by glancing through it. Every model has to be considered on case-by-case basis and appropriate test data should be used for quantification of errors.

6 CONCLUSION AND RECOMMENDATION

6.1 Conclusion

As we can see, using mathematical models will lead to different results if the assumptions are made. A careful identification of the component values will give precise results. But it all depends on the field that those results would be used. For example, for educational purposes, the spring could be assumed to have infinite stretching distance, as the result of the calculations would not be applied in any real life models, therefore the error that it would produce is not of significant importance. The model would be good enough to give people an understanding of how the systems works and simulation of the system would add further effect for comprehension. In case the mathematical model is used to test a drug for medical purposes, where the standard precision is set to 99%, the variables should be set more accurately and it is highly desirable to avoid assumptions, unless their effect is negligible. Same applies for the space applications, where the models and calculations are very complex, and a small error may result in damage to the equipment or even human life loss, all the variables and inputs should be precisely calculated, and only then would the mathematical model give actual behaviour of the systems in the real life and the desirable effect. Perhaps the most important advantage of using a mathematical model is that due to computer science advancement it is considerably easier to make simulations with the aid of computers. Computer simulations are used for wide range of applications, starting from simple educational programs up to really complex medical or space applications. Not only mathematical models make it easier to design systems, they also reduce cost of the project by eliminating the need to construct the model and test for their behaviour with different input variables and working environment. As every method has its weaknesses, the main disadvantage of using a mathematical model is that it could be complicated to identify the behaviour of the system and formulize it for computer to process. It may happen so that the phenomenon we are working on does not follow exactly the model we have selected to test it. And as was mentioned before, assumptions could lead to a considerable error and deviation from the actual results. Therefore, if the efficiency of the system of concern, the input variable and the criteria under which the system works should be carefully identified and included as parameters in the mathematical model we are using to test it. Another main question that arise

is how do we know which model is reliable? Unfortunately there is no simple answer. The model cannot be decided to be appropriate by just looking at it. Nobody can glance the model and tell whether it is a good approximation of the application. In case the model is simple, like Ohm's law, where only parameters are Voltage, Current and Resistance, $I = \frac{V}{R}$, where Current is directly proportional to the ratio of Voltage and Resistance. Here no additional information is needed, the model represents the behaviour fully, and no assumptions are made. But what if our model is much more complex than that. What if, the model we are using is of third order, where the rest of the equation has been truncated, and then in order to further simplify the model, a second order approximation of the model was taken. In this case, the model cannot be decided to be reliable just by looking at it. As it was proved by DC motor models, every model has to be considered on case-by-case basis. There is no single right approach for every models, and the test data has clearly shown how same parameters introduced an error in one example and had no effect in the second one. First, the dependent variable has to be identified, and then every one of them must be tested with test data to see how they influence the outcome of the model. How do we test the model for reliability then? A systematic approach must be taken. Here are the steps that should be taken into consideration:

1. Does the behavior of the system exactly represented by the mathematical model selected to study or for simulation.
2. The assumptions made should be identified and considered in the model as additional control parameters.
3. The error coefficient should be calculated for large scale applications and considered if it is acceptable or not.
4. If the accuracy of the model is of big importance, then tests should be run to compare simulation results with experimental results, if they are not equal, parameters of the model should be reconsidered until appropriate results are obtained.
5. Every dependent parameter must be identified, and separately tested for their effect on the results.

Only after these steps are taken, the conclusion about the model can be made. Perhaps the best advantage of using a mathematical model is that the accuracy can be set by user, and it is up to

user to decide how accurate he wants his model to be. The usage of the mathematical models in the modern science is not only desirable for sake of simplicity, but also reliability as they would produce the most accurate results possible. Since mathematical models are only tools, reliability of the model depends on its performance as well as how carefully it is used. As the computer technology advances and the general tendency shifts to computer usage, it is quite impossible to avoid using mathematical models, even though careless applications could result in error, which sometimes could lead to dramatic mistakes.

6.2 Recommendations

In this project we have covered topics about mathematical models, how they are used in engineering fields and other industries. We discussed the shortcomings of the models and how they should be avoided. We also tested several models for their performance and the obtained results were displayed for visual analysis. However, this project can further be improved by simulating more models, in order to have a deeper understanding on how models behave under given circumstances. In order to accomplish this, a more systematic approach must be taken. First, models should be grouped into different categories and then, depending on the categories, tests must be prepared for each category. This will give a better understanding on how every type of model behaves, the relative error that they introduce, which will help to utilize similar models by stereotyping, knowing the shortcomings of the model. However, as was shown in this research, each model is unique, and the application of the model plays an important role, so having the idea of how model behaves will not solve all the problems. If user feels that there is a need for additional tests, these tests should be conducted to have the data of models performance. The data will show errors due to assumptions, due to truncation, due to reducing the order of complexity and due to linearization if used. Other tests can be designed for special cases if necessary.

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